Risk indicators Calculation

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Notations

$N+1$	number of business days in strategy, benchmark and risk-free assets (historical data size)
$i \in \{0, .., N\}$	business day position
n	computation period length in business day (so used data size $= n+1$)
n_T	computation period length in time unity
N_T	number of business days per time unity in sliding computations (e.g. 21 for monthes and 252 for years)
k	computation day position in $\{1, .., N\}$
k_0	lower bound of computation period (so $k_0 = k - n$)
$\{T_1, .., T_M\}$	sorted set of the complete periods covering the historical period (assuming that a first incomplete period T_0 exists), for example : - monthly : $\{T_1, .., T_M\} \subset \{199\cdot, .., 200\cdot\} \times \{\text{Jan}, .., \text{Dec}\}$ - two-monthly : $\{T_1, .., T_M\} \subset \{199\cdot, .., 200\cdot\} \times \{\text{Jan-Feb}, .., \text{Nov-Dec}\}$ - yearly : $\{T_1, .., T_M\} \subset \{199\cdot, .., 200\cdot\}$
$K(T)$	position in $\{0, .., N\}$ of the last day of the time unity T

Thus

- in the sliding case, $n = n_T * N_T$ and $k \in \{n, .., N\}$
- in the calendar case, $k = K(T_1), .., K(T_M)$ and $k_0 = K(T_0), .., K(T_{M-1})$, so $n = k - k_0$
- in the snapshot case, $M = 1$, T_1 is such that $K(T_1)$ is the last day of a complete month and $k_0 = K(T_{M-1})$ is the last day of the month preceding the computation period.

$S(i)$	strategy spot at business day i
$B(i)$	benchmark spot at business day i
$R(i)$	risk-free asset spot at business day i
$\Delta_{i,j}$	difference (in days) between **dates** correponding to business days i and j
$r_{i,j}(X) = \frac{X(j)}{X(i)} - 1$	return of asset X between day i and day j
$\ell_{i,j}(X) = \ln \frac{X(j)}{X(i)}$	logarithmic return of asset X between day i and day j

1 Definition of indicators

In order to perform annualizations in return and volatility computations in the calendar case, we choose to define the measure in years of the computation period as $y_k = \frac{\Delta_{k_0,k}}{365}$. In the sliding case, we choose to simply divide the computation period length by a given year size expressed in business days, e.g. $y_k = n/252$. We will simply take this value in the following notations.

Thus, indicators are defined according to the time configuration (sliding or calendar).

1.1 Return

The return corresponds to the change in percentage in the value of an investment over an evaluation period. AMF recommends to annualize returns only for computation periods bigger than one year :

$$r_k(S) = \begin{cases} r_{k_0,k}(S) & \text{if} \quad y_k \leq 1 \\ [1 + r_{k_0,k}(S)]^{\frac{1}{y_k}} - 1 & \text{if} \quad y_k > 1 \end{cases}$$

1.2 Volatility

Volatility is a measure of an investment propensity to go up and down in price. A volatile investment is one that has a tendency to move sharply through a wide price range. Mathematically, this is expressed as the average standard deviation of daily price change from the average.

$$\sigma_k(S) = \sqrt{\frac{1}{y_k}\frac{n}{n-1}\sum_{i=k_0+1}^{k}\Big(\ell_{i-1,i}(S) - \mathbb{E}_k[\ell(S)]\Big)^2}$$

where $\mathbb{E}_k[\ell(X)] = \frac{1}{n}\sum_{i=k_0+1}^{k} \ell_{i-1,i}(X)$

1.3 Sharpe ratio

The Sharpe ratio is a measure, per unit of risk, of the excess return of an investment over the risk free rate. The Sharpe ratio is used to characterize how well the return of a strategy compensates the risk taken by the investor. The Sharpe ratio compares the difference of annualized returns of strategy and risk-free asset to the volatility (resp. down volatility) of the strategy :

$$\mathcal{S}_k^h(S,R) = \frac{\mathcal{R}_k(S) - \mathcal{R}_k(X)}{\sigma}\, 1_{\mathbb{R}^+} + \Big(\mathcal{R}_k(S) - \mathcal{R}_k(X)\Big)\, 1_{\mathbb{R}^-}$$

Remark : here the returns are **always** annualized (even if computation period length is lower than one year).

1.4 Beta

Beta is a measure of how the return of an investment is correlated to the return of its benchmark. A positive beta means that the investment generally follows its benchmark. Whereas a negative beta shows that the investment follows inversely the benchmark. It is the slope coefficient of the (least-square) straight line obtained by a linear regression on samples (B_i, S_i):

$$\beta_k(S,B) = \frac{\mathbb{C}_k(S,B)}{\sigma_k^2(B)}$$

1.5 Max drawdown

The max drawdown is the historical maximum loss in percentage over a time period that an investor would have faced if he had invested in a defined instrument at the highest price and sold at the lowest. This is the worst return of the period :

$$\mathcal{MDD}_k(S) = \min_{k_0+1 \leq i \leq k} \left(\frac{\min\limits_{i \leq j \leq k\}} S_j}{\max\limits_{k_0+1 \leq j \leq i\}} S_j} - 1 \right)$$

1.6 Time to recovery

The time to recovery is the longest period, after a drop in value; it takes an investment to return to its previous high. Writing $S_k^{\max}$ the maximum value appearing in the max drawdown, and $j_k^{\min}(S)$ the index in $\{k_0+1, .., k\}$ of the minimum one, the time to recovery can be defined as

$$\mathcal{TR}_k(S) = j_k^{\text{rec}} - j_k^{\min}$$

where $j_k^{\text{rec}} = \min \left\{ j \in \{j_k^{\min}, .., k\} \middle| S_j \geq S_k^{\max} \right\}$. We also need to measure this time in calendar weeks, by taking the difference between the related dates and dividing it by 7, before finally rounding to the closest integer value :

$$\mathcal{TR}_k^{\text{week}}(S) = \left\lfloor \frac{\Delta_{j_k^{\min}, j_k^{\text{rec}}}}{7} + \frac{1}{2} \right\rfloor .$$